

Pion and photon induced reactions on the nucleon in a unitary model.

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Abstract

We present a relativistic calculation of pion scattering, pion photoproduction and Compton scattering on the nucleon in the energy region of the Δ -resonance (upto 450 MeV photon lab energy), in a unified framework which obeys the unitarity constraint. It is found that the recent data on the cross section for nucleon Compton scattering determine accurately the parameters of the electromagnetic nucleon- Δ coupling. The calculated pion-photoproduction partial-wave amplitudes agree well with the recent Arndt analysis.

February 9, 2008

1 Introduction

It is well known that in Compton scattering from the nucleon, by using arguments based on unitarity and causality [1], strong constraints can be put on the cross section. These are usually formulated in terms of the fixed-t dispersion relations [2] expressing the real part of the six Hearn-Leader amplitudes [3] through the imaginary part. The latter is directly related by the optical theorem to the pion-photoproduction cross section. In such an approach the relatively small Born terms are added to account for the low-energy behaviour of the full amplitude. The slow convergence of the dispersion integrals requires several subtraction functions which are related to the t-channel singularities, especially to the $\pi\pi$ exchange [4]. Since the latter is poorly determined one has to use some models for the multiple meson exchange, thus making the approach model dependent. A lower unitary bound on the Compton cross section can be obtained by setting the real parts of the amplitude equal to zero [2] or to the Born contribution [5]. This of course allows to avoid the uncertainties of the dispersion approach, however the results obtained seem to agree with the data in the Δ -resonance region thus leaving no room for the proper real parts calculated from the dispersion integral. On the other hand in this approach the link with the decay properties of the nucleon resonances is getting obscured. In particular it is difficult to extract the contribution of the Δ -resonance to Compton scattering and parameters of the decay $\Delta \rightarrow N + \gamma$.

In order to express more clearly the contribution of the Δ -resonance, a relativistic tree-level calculation was performed[6] for nucleon Compton scattering. In this calculation the Δ -resonance was included via the Rarita-Schwinger propagator with a complex self energy to account for its pion decay width and thus, implicitly, for the pion channels. The calculation showed that even at energies near the pion threshold, $E_\gamma \approx 150$ MeV, the contribution of the Δ -resonance is crucial. Even though a good agreement with the data was obtained, one aspect missing in the calculation is that unitarity is obeyed only approximately.

From calculations on pion photoproduction (see, e.g.,[7, 8]) it is known that the unitarity constraint, which can for example be imposed via Watson's theorem connecting (π, π) and (γ, π) amplitudes, is crucial to obtain the correct interference between the resonant and the background contributions. In this work we have improved on the calculation of ref.[6] in this respect.

2 Outline of the model

A simple approach that is particularly suited for imposing the unitarity constraint while keeping at the same time a direct link to the basic Feynman diagrams, is the K -matrix approach. In this approach the T-matrix, $S = 1 + 2iT$, is represented as $T = K/(1 - iK)$, from which it is evident that the scattering matrix S is unitary when the K -matrix is hermitian.

We have employed the K -matrix approach in the space $\pi N \oplus \gamma N$. In this

way Compton scattering is investigated together with the pion-nucleon scattering and pion photoproduction. This has the additional advantage that all three processes are calculated *consistently* which puts stronger constraints on the model parameters.

Due to time-reversal invariance the partial wave K -matrix will be a real and symmetric 4×4 matrix in a basis spanned by two pion-nucleon channels, corresponding to different values for the total πN isospin ($1/2$ and $3/2$) and two photon-nucleon channels, corresponding to different helicities (or, equivalently electric and magnetic radiation). For the partial wave decomposition we use the Jacob–Wick formalism as given in appendices of refs. [4, 9].

Within the model space unitarity is satisfied exactly, however the two pion channel, which is known to become important at energies in excess of 400 MeV, is not included. We find that though the phases in the partial wave amplitudes for Compton scattering off the proton have changed considerably as compared to the non-unitarized calculation[6], the cross sections have changed little in the energy region upto the Δ -resonance. At the resonance and beyond there are substantial differences.

The K -matrix is approximated by the sum of tree-level diagrams including direct (s -type) and crossed (u -type) 'driving' terms with intermediate nucleon, N^* (Roper)- and Δ -resonances with *real* self energies equal to the mass of the resonances. In the t -channel for pion scattering, σ - and ρ -meson exchanges are taken into account. In pion photoproduction the π -, ρ - and ω -meson are included in the t -channel, where the latter two have only a marginal effect on the calculated quantities and the first is necessary to ensure current conservation. In Compton scattering only π^0 -meson exchange is included.

The K -matrix formalism results from the Bethe-Salpeter equation in the approximation that the principal value of the loop integrals is neglected and only the contribution from the discontinuity is kept. Stated differently, the particles forming loops are taken to be on the mass shell. The width of the resonances is generated dynamically in the calculation of the T -matrix as a result of iteration of the direct diagrams. The pole contributions from the loops involving u - and t -type diagrams in the K -matrix give rise to the pion-loop vertex corrections to the πNN and γNN vertex functions. Both the decay width and vertex corrections are thus generated dynamically in the K -matrix approach. Because of this internal consistency the unitarity constraint is exactly satisfied. Two-pion channel is not included in the model space. Since the phase-space for this channel opens only gradually it can safely be ignored up to photon energies of about 400 MeV.

As mentioned only the pole contributions from the pion (and photon) loops are included and, for reasons of simplicity, the principle-value part of the full 4-dimensional integral is neglected. The effect of the latter has been studied in the framework of a relativistic integral equation in which the pion is restricted to its mass shell in Ref. [10] for the $\pi N \rightarrow \pi N$ and in Ref. [11] for $\gamma N \rightarrow \pi N$ reactions. It can however be argued [12] that the

effect of the real part of the loop integrals will be mainly a renormalization of the coupling constants and the masses of the particles involved. In our calculations these are taken to be the physical values where known and are otherwise treated as free parameters; we believe that such renormalizations are probably of small importance.

The Δ -state is included via the spin- $\frac{3}{2}$ Rarita-Schwinger propagator, which off-shell contains also a spin- $\frac{1}{2}$ background. In each of the vertices involving the Δ therefore also an off-shell coupling parameter enters, which determines the coupling to the spin- $\frac{1}{2}$ sector of the Rarita-Schwinger propagator. These off-shell couplings appear to be of crucial importance to reproduce the data. In the πNN coupling vertex we have allowed for a mixture of pseudo-scalar and pseudo-vector coupling specified by parameter χ ($\chi = 0$ corresponds to pure pseudo-vector coupling). For non-zero values of χ the Kroll-Rudermann term is included in pion photoproduction to restore gauge invariance.

3 Results and Discussion.

As emphasized, in the calculations the unitarity constraint is satisfied, even to higher orders in α , the fine structure constant. For πN scattering the higher order corrections in α are negligible and we will therefore discuss this case first to fix the pion coupling parameters.

Our model for πN scattering is very similar to that of Goudsmit et al.[12]. The main difference is that, since we are interested in somewhat higher energies, we have also included the Roper-resonance in the calculations. This improves the fit in the P_{11} channel as to be expected, but hardly influences any of the other partial-wave amplitudes. To investigate the effect of the strong couplings we have used the results of three different fits to the πN phase shifts. Two of these, parameter sets # 1 and # 2 in Table 1, have been taken from ref.[12]. These two parameter sets allow for the investigation of the effect of pseudo-scalar v.s. pseudo-vector πNN -coupling. To study the effect of the off-shell coupling in the $\pi N\Delta$ -vertex (characterized by the parameter z_π) we have analysed also the third set given in Table 1.

All three parameter sets given in Table 1 give a comparable overall fit to the Arndt partial-wave data[14]. Parameter set # 3 gives somewhat better results at higher energies in the S_{11} -channel but slightly worse results in the S_{31} -channel. Since for the first two parameter sets our results are very similar to those of ref.[12] we will not discuss these any further in this short letter.

The results of our calculations for the pion photoproduction partial-wave amplitudes and the Compton-scattering cross section are compared with the data in Figs. 1 and 2. In these calculations only the four parameters of the $\gamma N\Delta$ -vertex have been optimized. As shown in Table 2, there is a range of values for G_2 , compensated by an appropriate change in z_1 and z_2 , for which a comparably good fit can be obtained. A best fit to the data for Compton

set #	$g_{\pi NN}$	χ	$g_{\pi N\Delta}$	z_π	G_σ	$g_{\rho NN}$	κ_ρ
1	12.95	0.0	2.19	-.34	23	5.87	2.1
2	12.95	0.2	2.19	-.34	43	2.90	2.1
3	12.95	0.0	2.19	-.16	28	5.40	2.1

Table 1: Different sets of parameters used in the calculation of the pion-nucleon scattering. In the definition of the interaction Lagrangian ref. [12] is followed, only the PV/PS mixing parameter x is renamed to χ . Parameter sets # 1 and # 2 correspond to two fits to the πN scattering data as presented in ref. [12], at the extremes of their parameter spectrum. All parameters not explicitly mentioned are taken from this work with $g_{\pi\pi\rho} = 6.065$. The Roper resonance is included following ref. [13] with $H = 0.145$ and vanishing width. Only its one-pion partial decay width is generated dynamically.

set #	G_1	z_1	G_2	z_2	$R(E2/M1)$
1	4.3	0.15	2.0	4.0	-4.47%
2	4.3	0.05	4.0	2.5	-2.56%
3	4.3	0.0	6.0	1.8	-0.57%

Table 2: Different sets of parameters for the $\gamma N\Delta$ vertex that give a comparable fit to the cross-section data. The parameters for the ω meson couplings are taken from ref. [13]. For the electro-magnetic decay of the Roper resonance we used [13] $G_p = -0.544$ and $G_n = +0.552$. The $E2/M1$ decay ratio is defined according to Eq. (1).

scattering is obtained with parameter set # 1 from Table 2.

A parameter often quoted for the $N\Delta\gamma$ -vertex is the E2/M1 ratio for the electromagnetic decay of the Δ -resonance. This ratio is however not directly related to physical observables due to background contributions. It is defined as the ratio of the electric and magnetic decay rate of an 'on-shell' Δ -resonance. As such it depends only on the parameters G_1 and G_2 and *not* on the off-shell coupling parameters z_1 and z_2 [7],

$$R\left(\frac{E2}{M1}\right) = -\frac{2G_1 - G_2 \frac{M_\Delta}{M}}{2G_1 \frac{3M_\Delta + M}{M_\Delta - M} - G_2 \frac{M_\Delta}{M}} \quad (1)$$

where M (M_Δ) is the nucleon (Δ -resonance) mass. The predictions for R , as given in Table 2, thus vary strongly with G_2 while keeping agreement with the data where the variation of G_2 is compensated with a variation of the off-shell parameters.

In pion photoproduction the partial wave amplitudes are largely insensitive to the off-shell parameters. From Figs. 1 and 2 one can see that the

different choices for G_2 affect only the $E_{1+}^{3/2}$ multipole amplitude¹ in pion photoproduction at energies exceeding 300 MeV. None of the other partial wave amplitudes are affected.

In Compton scattering a variation of G_2 mainly affects the f_{EE}^{2-} and the f_{ME}^{1+} amplitudes which are related to the $E_{1+}^{3/2}$ amplitude in the pion photoproduction channel. The $\gamma N\Delta$ vertex enters the amplitude quadratically and the strong vertices do not enter explicitly. Therefore Compton scattering is strongly dependent on the off-shell coupling to the background spin- $\frac{1}{2}$ fields. The corresponding off-shell coupling parameters are thus determined rather accurately. This observation goes in line with the finding [18, 12] for the pion-nucleon scattering where a strong dependence on the off-shell parameter z_π in the πNN vertex has been obtained.

None of the different choices for the parameters for πN scattering given in Table 1 do noticeably affect the results for Compton scattering. Including a pseudo-scalar coupling improves the agreement with the π -N-scattering data at higher energies but has hardly any influence on the pion photoproduction channel. There is a considerable sensitivity to the off-shell coupling parameter in the $\pi N\Delta$ -vertex, z_π . As mentioned, the fit for the S_{31} πN phase shift is somewhat worse for parameter set # 3 in Table 1, but the agreement for pion photoproduction is considerably improved, especially for the $M_{1+}^{1/2}$ channel. Again, the Compton scattering calculations are not affected.

For energies well below the Δ -resonance the unitarity constraint is not very important when calculating the Compton cross section. This is shown in Fig. 3 where the results of the present calculations are compared with the non-unitarized tree approximation where a finite width for the Δ -resonance is included[6]. Only at energies where the Δ -resonance is dominating the spectrum the differences are appreciable.

In this figure also the result of a calculation is shown where the photon-decay of the Δ is switched off. This shows clearly the importance of the Δ even at energies near the pion threshold. Only due to a strong destructive interference at small angles and a strong constructive interference at large angles of the nucleon- and the Δ -amplitudes the characteristic vanishing of the cross section at 0 degrees can be accounted for.

In conclusion, Compton scattering is demonstrated to be most suitable for determining the parameters of the $\gamma N\Delta$ vertex since the Compton amplitude is largely *insensitive* to the strong channels, while it is very *sensitive* to the photon- Δ coupling. Quite the opposite is found for pion photoproduction. We have shown that in a rigorously unitary calculation of Compton scattering from the proton the interference of the nucleon and Δ -isobar am-

¹The notation $\mathcal{M}_{L\pm}^I$ is used in pion photoproduction where \mathcal{M} stands for the electric ($\mathcal{M} = E$) or magnetic ($\mathcal{M} = M$) type of the photon with the final πN state characterized by the orbital angular momentum L , parity $p = (-1)^{L+1}$, total angular momentum $J = L \pm 1/2$ and total isospin $I = 1/2, 3/2$. In pion scattering $L_{21\,2J}$ is used. In Compton scattering the notation is $f_{\mathcal{M}\mathcal{M}'}^{L\pm}$ where the total angular momentum is $J = L \pm 1/2$ and parity is $p = (-1)^L$ for $\mathcal{M} = E$ or $p = (-1)^{L+1}$ for $\mathcal{M} = M$.

plitudes is crucial to understand the observed structure in the cross section as was also found in a tree level calculation[6]. The unitary constraint turns out to be important in the calculation of the cross section at photon energies exceeding 250 MeV.

One of us (A.Yu. K.) thanks the staff of the KVI for the kind hospitality extended to him at the KVI Groningen. He would also like to thank the Netherlands Organization for International Cooperation in Higher Education (Nuffic) and the Nederlandse organisatie voor Wetenschappelijk Onderzoek (NWO) for financial support during his stay in the Netherlands.

This work was financially supported by de Stichting voor Fundamenteel Onderzoek der Materie (FOM).

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fig. 1 The pion-photoproduction partial-wave amplitudes are compared with the analysis of ref.[15]. In particular it is shown that the effect of changing G_2 (Table 1, set # 1 and Table 2, set # 3) is small as compared to the norm calculation (Table 1, set # 1 and Table 2, set # 1). The off-shell parameter in the $N\Delta\gamma$ -vertex, z_π , is much larger as shown in the third calculation using (Table 1, set # 3 and Table 2, set # 1).

fig. 2 A comparison of different calculations for Compton scattering the parameters used are the same as for figure 1. The data for the Compton-scattering cross section are from ref. [16, 17]

fig. 3 The results of the full ('Norm' calculation of figures 1 and 2) calculation for the Compton cross section is compared with a calculation in which the coupling of the photon to the Δ -resonance is set to zero ('No Delta') and with a calculation along the lines of ref.[6] using the same values for the parameters in the photon coupling vertices ('No Unitarity').